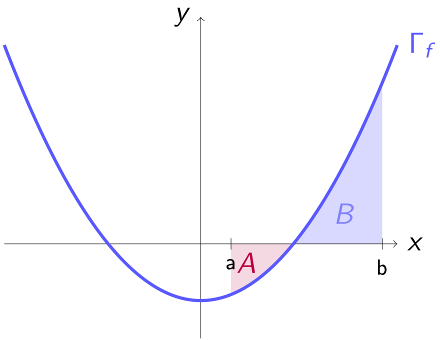
# 5.2. Određeni integral

11. 12. 2020.

Fotografija: Jeremy Thomas

# Određeni integral




$$\int_a^b f(x)\,\mathrm{d}x := P(B) - P(A).$$

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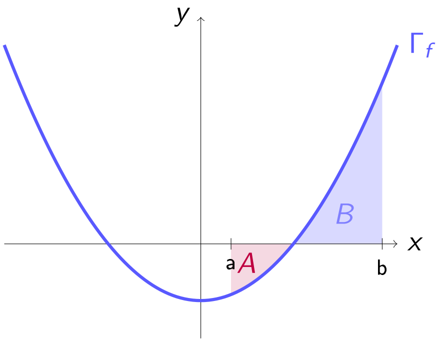


$$\int_a^b f(x)\,\mathrm{d}x := P(B) - P(A).$$

- Ako je $a < b$,

$$\int_b^a := -\int_a^b .$$

# Teorem (Newton-Leibnizova formula)

Ako je funkcija $f : D \subseteq \mathbb{R} \to \mathbb{R}$ neprekidna na nekom segmentu $[a, b] \subseteq D$, i ako je $F$ bilo koja antiderivacija funkcije $f$, tada je

$$\int_a^b f(x)\,\mathrm{d}x = \underbrace{F(x)\Big|_a^b}_{\substack{\text{prirast}\\ \text{funkcije}\\ F \text{ od } a \text{ do } b}} := F(b) - F(a).$$

# Zadatak 46(a)

Izračunajte integral

$$\int_1^2 x^{-3}\,\mathrm{d}x.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

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Izračunajte integral

$$\int_1^2 x^{-3}\,\mathrm{d}x.$$

*Rješenje.* Imamo

$$\int_1^2 x^{-3}\,\mathrm{d}x = \frac{x^{-2}}{-2}\bigg|_1^2$$

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Izračunajte integral

$$\int_1^2 x^{-3}\,\mathrm{d}x.$$

*Rješenje.* Imamo

$$\int_1^2 x^{-3}\,\mathrm{d}x = \frac{x^{-2}}{-2}\Big|_1^2$$

$$= \frac{2^{-2}}{-2} - \frac{1^{-2}}{-2}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

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$$\int_1^2 x^{-3}\,\mathrm{d}x.$$

*Rješenje.* Imamo

$$\begin{aligned}\int_1^2 x^{-3}\,\mathrm{d}x &= \frac{x^{-2}}{-2}\Big|_1^2\\ &= \frac{2^{-2}}{-2} - \frac{1^{-2}}{-2}\\ &= -\frac{1}{8} + \frac{1}{2}\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

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$$\begin{aligned}\int_1^2 x^{-3}\,\mathrm{d}x &= \frac{x^{-2}}{-2}\Big|_1^2\\ &= \frac{2^{-2}}{-2} - \frac{1^{-2}}{-2}\\ &= -\frac{1}{8} + \frac{1}{2}\\ &= \frac{3}{8}.\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

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# Zadatak 46(b)

Izračunajte integral $\int_1^{64} x^{\frac{5}{6}}\,\mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
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*Rješenje.* Imamo

$$\int_1^{64} x^{\frac{5}{6}}\,\mathrm{d}x = \left.\frac{x^{\frac{11}{6}}}{\frac{11}{6}}\right|_1^{64} = \left.\frac{6}{11}x^{\frac{11}{6}}\right|_1^{64}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

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*Rješenje.* Imamo

$$\int_1^{64} x^{\frac{5}{6}}\,\mathrm{d}x = \left.\frac{x^{\frac{11}{6}}}{\frac{11}{6}}\right|_1^{64} = \left.\frac{6}{11}x^{\frac{11}{6}}\right|_1^{64} = \frac{6}{11}\left(64^{\frac{11}{6}} - 1^{\frac{11}{6}}\right)$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

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Izračunajte integral $\int_1^{64} x^{\frac{5}{6}}\,\mathrm{d}x$.

*Rješenje.* Imamo

$$\int_1^{64} x^{\frac{5}{6}}\,\mathrm{d}x = \left.\frac{x^{\frac{11}{6}}}{\frac{11}{6}}\right|_1^{64}$$
$$= \left.\frac{6}{11}x^{\frac{11}{6}}\right|_1^{64}$$
$$= \frac{6}{11}\left(64^{\frac{11}{6}} - 1^{\frac{11}{6}}\right)$$
$$= \frac{6}{11}\left(\left(2^{\cancel{6}}\right)^{\frac{11}{\cancel{6}}} - 1\right)$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
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$$\int \mathrm{d}x = x + C$$

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# Zadatak 46(c)

Izračunajte integral

$$\int_0^1 \left(\sqrt{2x} - 3\sqrt{x}\right)\,\mathrm{d}x.$$

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*Rješenje.* Imamo

$$\int_0^1 \left(\sqrt{2x} - 3\sqrt{x}\right)\, \mathrm{d}x = \int_0^1 \left(\sqrt{2} - 3\right) x^{\frac{1}{2}}\, \mathrm{d}x$$

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*Rješenje.* Imamo

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$$= \left(\sqrt{2} - 3\right) \cdot \frac{2}{3} x^{\frac{3}{2}} \Big|_0^1$$

$$\int \mathrm{d}x = x + C$$

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$$= \left(\sqrt{2} - 3\right) \cdot \frac{2}{3} x^{\frac{3}{2}} \Big|_0^1$$

$$= \left(\sqrt{2} - 3\right) \cdot \frac{2}{3} \left(1^{\frac{3}{2}} - 0^{\frac{3}{2}}\right)$$

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$$= \left(\sqrt{2} - 3\right) \cdot \frac{2}{3} x^{\frac{3}{2}} \Big|_0^1$$

$$= \left(\sqrt{2} - 3\right) \cdot \frac{2}{3} \left(1^{\frac{3}{2}} - 0^{\frac{3}{2}}\right)$$

$$= \left(\sqrt{2} - 3\right) \cdot \frac{2}{3}$$

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$$\int \frac{\mathrm{d}x}{1 + x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

# Zadatak 46(c)

Izračunajte integral

$$\int_0^1 \left(\sqrt{2x} - 3\sqrt{x}\right)\,\mathrm{d}x.$$

*Rješenje.* Imamo

$$\int_0^1 \left(\sqrt{2x} - 3\sqrt{x}\right)\,\mathrm{d}x = \int_0^1 \left(\sqrt{2} - 3\right) x^{\frac{1}{2}}\,\mathrm{d}x$$

$$= \left(\sqrt{2} - 3\right) \cdot \frac{2}{3} x^{\frac{3}{2}} \Big|_0^1$$

$$= \left(\sqrt{2} - 3\right) \cdot \frac{2}{3} \left(1^{\frac{3}{2}} - 0^{\frac{3}{2}}\right)$$

$$= \left(\sqrt{2} - 3\right) \cdot \frac{2}{3}$$

$$= \frac{2\sqrt{2}}{3} - 2.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1 - x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1 + x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

# Zadatak 46(d)

Izračunajte integral $\int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{1+\cos(2x)}\,\mathrm{d}x.$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

# Zadatak 46(d)

Izračunajte integral $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{1+\cos(2x)}\,\mathrm{d}x.$

*Rješenje.* $$\int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{1+\cos(2x)}\,\mathrm{d}x = \int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{\cancel{1} + (2\cos^2 x \cancel{-1})}\,\mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a>0,\ a\neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a>0)$$

# Zadatak 46(d)

Izračunajte integral $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{1+\cos(2x)}\,\mathrm{d}x$.

*Rješenje.*
$$\int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{1+\cos(2x)}\,\mathrm{d}x = \int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{\cancel{1} + (2\cos^2 x \cancel{-1})}\,\mathrm{d}x$$
$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(1 - \frac{5}{\cos^2 x}\right)\mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

# Zadatak 46(d)

Izračunajte integral $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{1+\cos(2x)}\,\mathrm{d}x$.

*Rješenje.*
$$\int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{1+\cos(2x)}\,\mathrm{d}x = \int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{\cancel{1} + (2\cos^2 x \cancel{-1})}\,\mathrm{d}x$$
$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(1 - \frac{5}{\cos^2 x}\right)\mathrm{d}x$$
$$= \frac{1}{2}\left(x - 5\operatorname{tg} x\right)\Big|_0^{\frac{\pi}{4}}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

# Zadatak 46(d)

Izračunajte integral $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{1+\cos(2x)}\,\mathrm{d}x.$

*Rješenje.*

$$\int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{1+\cos(2x)}\,\mathrm{d}x = \int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{\cancel{1} + (2\cos^2 x \cancel{-1})}\,\mathrm{d}x$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(1 - \frac{5}{\cos^2 x}\right)\mathrm{d}x$$

$$= \frac{1}{2}(x - 5\,\mathrm{tg}\,x)\Big|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\left(\frac{\pi}{4} - 5\,\mathrm{tg}\,\frac{\pi}{4}\right) - \frac{1}{2}(0 - 5\,\mathrm{tg}\,0)$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \mathrm{tg}\,x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\mathrm{ctg}\,x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \mathrm{arctg}\,x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a} + C \quad (a > 0)$$

# Zadatak 46(d)

Izračunajte integral $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{1+\cos(2x)}\,\mathrm{d}x.$

*Rješenje.*

$$\int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{1+\cos(2x)}\,\mathrm{d}x = \int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{\cancel{1} + (2\cos^2 x \cancel{-1})}\,\mathrm{d}x$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(1 - \frac{5}{\cos^2 x}\right)\mathrm{d}x$$

$$= \frac{1}{2}(x - 5\operatorname{tg} x)\Big|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\left(\frac{\pi}{4} - 5\operatorname{tg}\frac{\pi}{4}\right) - \frac{1}{2}(0 - 5\operatorname{tg} 0)$$

$$= \frac{1}{2}\left(\frac{\pi}{4} - 5\cdot 1\right) - \frac{1}{2}(0 - 5\cdot 0)$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

# Zadatak 46(d)

Izračunajte integral $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{1+\cos(2x)}\,\mathrm{d}x$.

*Rješenje.*

$$\int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{1+\cos(2x)}\,\mathrm{d}x = \int_0^{\frac{\pi}{4}} \frac{\cos^2 x - 5}{\cancel{1} + (2\cos^2 x \cancel{-1})}\,\mathrm{d}x$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(1 - \frac{5}{\cos^2 x}\right)\mathrm{d}x$$

$$= \frac{1}{2}(x - 5\,\mathrm{tg}\,x)\Big|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\left(\frac{\pi}{4} - 5\,\mathrm{tg}\,\frac{\pi}{4}\right) - \frac{1}{2}(0 - 5\,\mathrm{tg}\,0)$$

$$= \frac{1}{2}\left(\frac{\pi}{4} - 5\cdot 1\right) - \frac{1}{2}(0 - 5\cdot 0)$$

$$= \frac{\pi}{8} - \frac{5}{2}.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \mathrm{tg}\,x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\,\mathrm{ctg}\,x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \mathrm{arctg}\,x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a} + C \quad (a > 0)$$

# Zadatak 46(e)

Izračunajte integral $\int_{7}^{7+2\pi} \sin x \, \mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \, \mathrm{d}x = e^x + C$$
$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \; a \neq 1)$$
$$\int \cos x \, \mathrm{d}x = \sin x + C$$
$$\int \sin x \, \mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

# Zadatak 46(e)

Izračunajte integral $\int_7^{7+2\pi} \sin x \, \mathrm{d}x$.

*Rješenje. 1. način.* Imamo

$$\int_7^{7+2\pi} \sin x \, \mathrm{d}x = -\cos x \Big|_7^{7+2\pi}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

# Zadatak 46(e)

Izračunajte integral $\displaystyle\int_7^{7+2\pi} \sin x \,\mathrm{d}x$.

*Rješenje. 1. način.* Imamo

$$\int_7^{7+2\pi} \sin x \,\mathrm{d}x = -\cos x \Big|_7^{7+2\pi}$$
$$= -\cos(7+2\pi) - (-\cos 7)$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \,\mathrm{d}x = e^x + C$$
$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x \,\mathrm{d}x = \sin x + C$$
$$\int \sin x \,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

# Zadatak 46(e)

Izračunajte integral $\int_7^{7+2\pi} \sin x \, \mathrm{d}x$.

*Rješenje. 1. način.* Imamo

$$\begin{aligned}\int_7^{7+2\pi} \sin x \, \mathrm{d}x &= -\cos x \Big|_7^{7+2\pi} \\ &= -\cos(7+2\pi) - (-\cos 7) \\ &= -\cos 7 + \cos 7\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

# Zadatak 46(e)

Izračunajte integral $\int_7^{7+2\pi} \sin x \, \mathrm{d}x$.

*Rješenje. 1. način.* Imamo

$$\begin{aligned}\int_7^{7+2\pi} \sin x \, \mathrm{d}x &= -\cos x \Big|_7^{7+2\pi} \\ &= -\cos(7+2\pi) - (-\cos 7) \\ &= -\cos 7 + \cos 7 \\ &= 0.\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

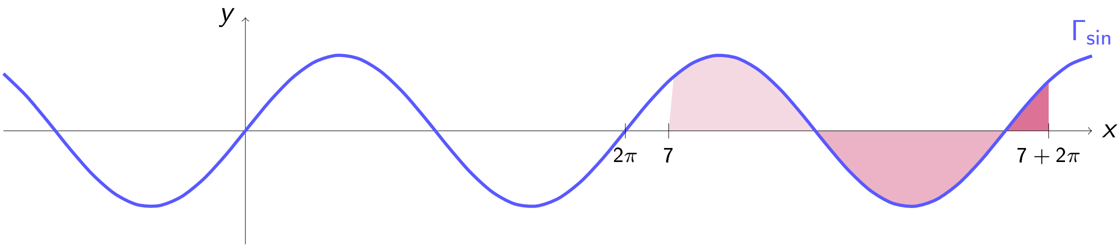
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

# Zadatak 46(e)

Izračunajte integral $\displaystyle\int_7^{7+2\pi} \sin x \, \mathrm{d}x.$

*Rješenje. 2. način.*

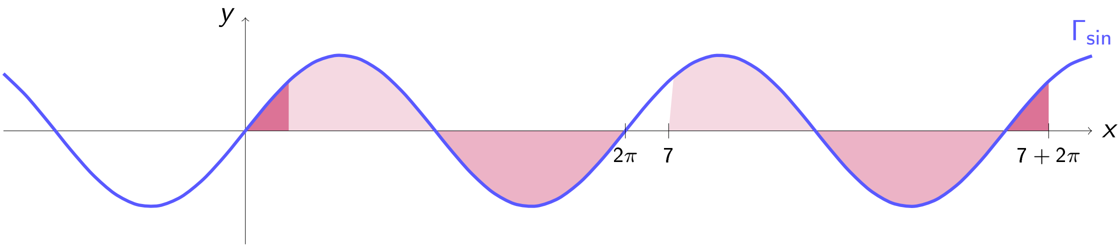



$$\int_7^{7+2\pi} \sin x \, \mathrm{d}x$$

# Zadatak 46(e)

Izračunajte integral $\displaystyle\int_7^{7+2\pi} \sin x \,\mathrm{d}x$.

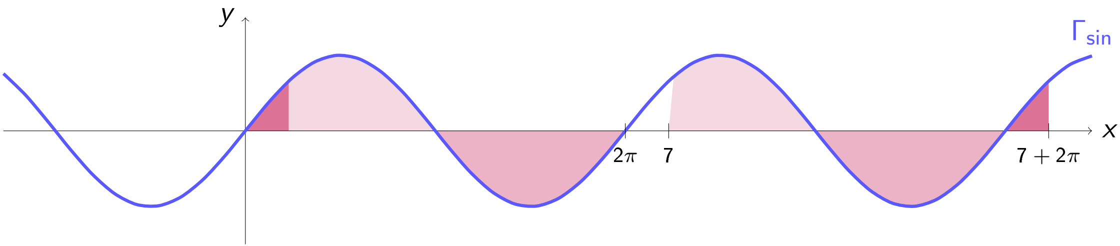
*Rješenje. 2. način.*




$$\int_7^{7+2\pi} \sin x \,\mathrm{d}x \overset{\text{slika}}{=} \int_0^{2\pi} \sin x \,\mathrm{d}x$$

Izračunajte integral $\displaystyle\int_7^{7+2\pi} \sin x \, \mathrm{d}x$.

*Rješenje. 2. način.*

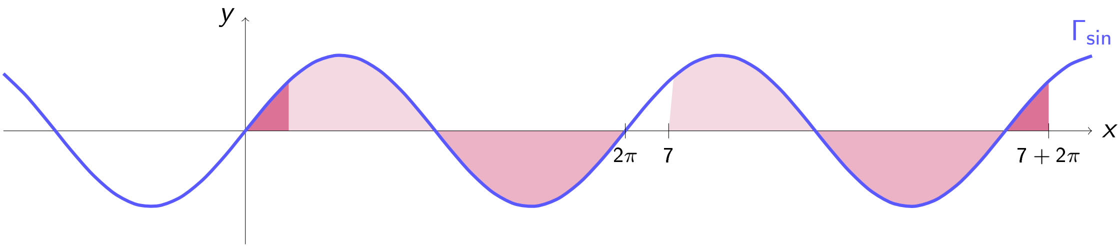



$$\int_7^{7+2\pi} \sin x \, \mathrm{d}x \overset{\text{slika}}{=} \int_0^{2\pi} \sin x \, \mathrm{d}x = -\cos x \Big|_0^{2\pi}$$

# Zadatak 46(e)

Izračunajte integral $\int_7^{7+2\pi} \sin x \, \mathrm{d}x$.

*Rješenje. 2. način.*

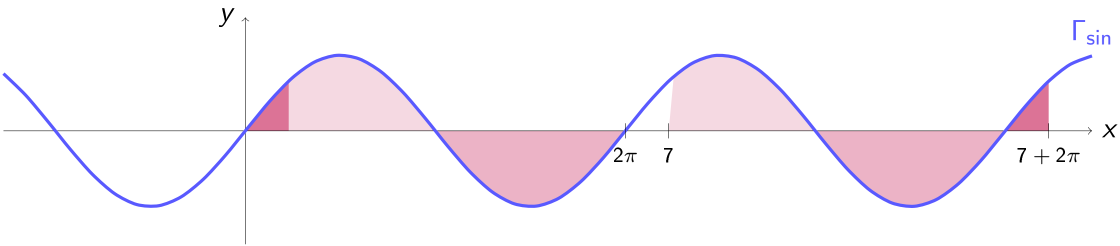



$$\int_7^{7+2\pi} \sin x \, \mathrm{d}x \overset{\text{slika}}{=} \int_0^{2\pi} \sin x \, \mathrm{d}x = -\cos x \Big|_0^{2\pi} = -\cos(2\pi) - (-\cos 0)$$

## Zadatak 46(e)

Izračunajte integral $\displaystyle\int_7^{7+2\pi} \sin x \, \mathrm{d}x.$

*Rješenje. 2. način.*

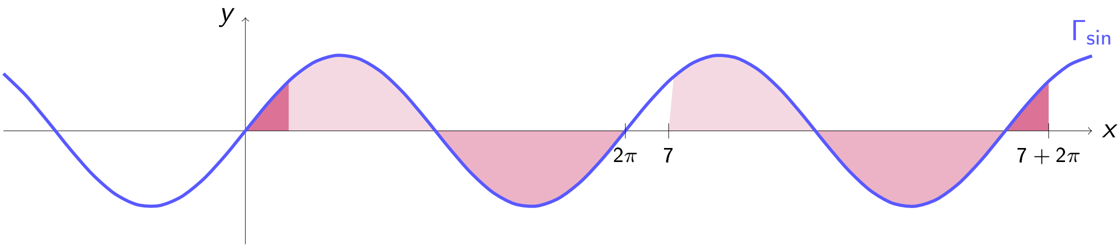



$$\int_7^{7+2\pi} \sin x \, \mathrm{d}x \overset{\text{slika}}{=} \int_0^{2\pi} \sin x \, \mathrm{d}x = -\cos x \Big|_0^{2\pi} = -\cos(2\pi) - (-\cos 0) = -1 + 1$$

# Zadatak 46(e)

Izračunajte integral $\int_7^{7+2\pi} \sin x \, \mathrm{d}x$.

*Rješenje. 2. način.*




$$\int_7^{7+2\pi} \sin x \, \mathrm{d}x \overset{\text{slika}}{=} \int_0^{2\pi} \sin x \, \mathrm{d}x = -\cos x \Big|_0^{2\pi} = -\cos(2\pi) - (-\cos 0) = -1 + 1 = 0.$$

# Korisna napomena

Ako je $f : \mathbb{R} \to \mathbb{R}$ neprekidna i periodična s periodom $T$, tada je

$$\int_a^{a+T} f(x)\,\mathrm{d}x = \int_b^{b+T} f(x)\,\mathrm{d}x \qquad \text{za sve } a, b \in \mathbb{R}.$$

# Zadatak 46(f)

Izračunajte integral $\int_{-2}^{2} x^7 \, \mathrm{d}x$.

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Izračunajte integral $\int_{-2}^{2} x^7 \, \mathrm{d}x$.

*Rješenje.* $\int_{-2}^{2} x^7 \, \mathrm{d}x = \frac{x^8}{8} \Big|_{-2}^{2}$

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*Rješenje.* $\int_{-2}^{2} x^7 \, \mathrm{d}x = \frac{x^8}{8}\Big|_{-2}^{2} = \frac{2^8}{8} - \frac{(-2)^8}{8}$

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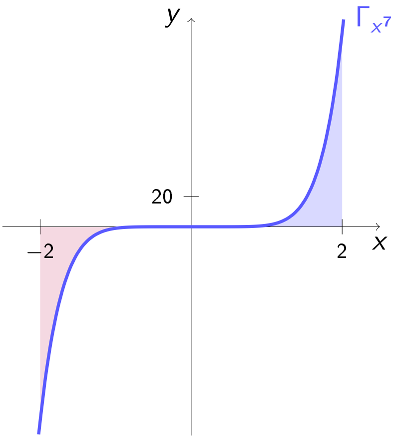

# Jako korisna napomena

Ako je $f : \mathbb{R} \to \mathbb{R}$ neprekidna i neparna, tada je

$$\int_{-a}^{a} f(x)\,\mathrm{d}x = 0 \qquad \text{za svaki } a \in \mathbb{R}.$$

# Zadatak 46(g)

Izračunajte integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, \mathrm{d}x$.

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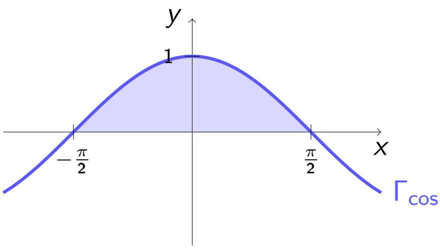

# Zadatak 46(g)

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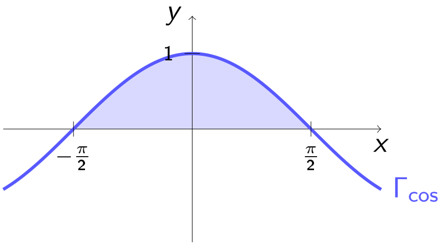
*Rješenje.*




Imamo

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \,\mathrm{d}x \overset{\text{slika}}{=} 2\int_{0}^{\frac{\pi}{2}} \cos x \,\mathrm{d}x$$

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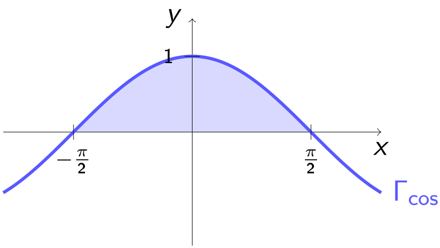


Imamo

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, \mathrm{d}x \overset{\text{slika}}{=} 2 \int_{0}^{\frac{\pi}{2}} \cos x \, \mathrm{d}x$$

$$= 2 \cdot \sin x \Big|_{0}^{\frac{\pi}{2}}$$

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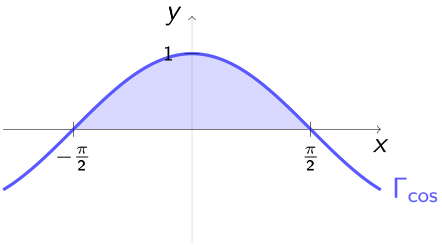
*Rješenje.*




Imamo

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \,\mathrm{d}x \overset{\text{slika}}{=} 2\int_{0}^{\frac{\pi}{2}} \cos x \,\mathrm{d}x$$

$$= 2 \cdot \sin x \Big|_{0}^{\frac{\pi}{2}}$$

$$= 2\left(\sin\frac{\pi}{2} - \sin 0\right)$$

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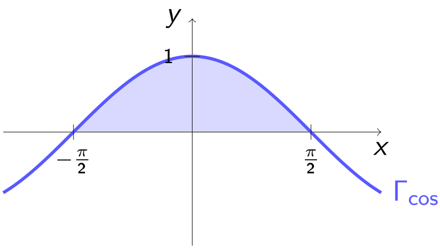
*Rješenje.*




Imamo

$$\begin{aligned}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, \mathrm{d}x &\overset{\text{slika}}{=} 2\int_{0}^{\frac{\pi}{2}} \cos x \, \mathrm{d}x \\ &= 2 \cdot \sin x \Big|_{0}^{\frac{\pi}{2}} \\ &= 2\left(\sin\frac{\pi}{2} - \sin 0\right) \\ &= 2.\end{aligned}$$

# Korisna napomena

Ako je $f : \mathbb{R} \to \mathbb{R}$ neprekidna i parna, tada je

$$\int_{-a}^{a} f(x)\,\mathrm{d}x = 2\int_{0}^{a} f(x)\,\mathrm{d}x \qquad \text{za svaki } a \in \mathbb{R}.$$